Published by Institute of Physics Publishing for SISSA/ISAS
Received: *November 22, 2002*
Accepted: *December 2, 2002*

On gravitational couplings in D-brane action

Angelos Fotopoulos[ab] **and Arkady A. Tseytlin**[ac*]

[a] *Theoretical Physics Group, Blackett Laboratory*
Imperial College, London SW7 2BZ, U.K.
[b] *Centre de Physique Theorique*
Ecole Polytechnique, 91128 Palaiseau, France
[c] *Smith Laboratory, The Ohio State University*
Columbus OH 43210, USA
E-mail: Angelos.Fotopoulos@cpht.polytechnique.fr,
tseytlin@mps.ohio-state.edu

Abstract: We compute the two closed string graviton — two open string scalar superstring scattering amplitude on the disc to show that there is no second-derivative curvature–scalar coupling term RX^2 in the low-energy effective action of a D-brane in curved space.

Keywords: D-branes, Superstrings and Heterotic Strings, AdS-CFT and dS-CFT Correspondence.

JHEP12(2002)001

*Also at Lebedev Physics Institute, Moscow

Contents

1. Introduction

The structure of gravitational coupling terms in the action for a D-brane probe moving in a curved space is of interest in many contexts, in particular, in brane-world constructions and gauge theory – string theory duality.

The standard Born-Infeld [1, 2] action for a D-brane [3, 4] in flat space [5] has a direct generalization to curved ambient space[1]

$$S_{DBI} = -T_{(p)} \int d^{p+1}\sigma e^{-\phi(X)} \sqrt{\det\left[(G_{\mu\nu}(X) + B_{\mu\nu}(X))\partial_\alpha X^\mu \partial_\beta X^\nu + F_{\alpha\beta}\right]} + \cdots . \quad (1.1)$$

Dots stand for various higher-derivative corrections (present already in flat space [6]). We shall ignore the well-known WZ-type couplings to R-R potentials and concentrate on the parity-even part of the action.

One may wonder still if the action (1.1) correctly describes the gravitational couplings of the D-brane even to the lowest order in derivatives. A way to determine the precise form of the low-energy D-brane effective action is to compare the corresponding vertices with the open–closed string S-matrix on the disc. Several of such studies checking the consistency of (1.1) and extending it to the next (4-th) derivative order were carried out in the past [7]–[13]. It was found (in agreement with expectation based on the leading gravitational interaction term on the disc in type-I string theory being R^2 [14]) that there is no Einstein-type R-term in the D-brane action in type-II superstring case[2] but there are $O(\alpha'^2)$ correction terms [11, 13, 12]

$$L^{(p)} = T_{(p)}\sqrt{G}e^{-\phi}\bigg(1 - \frac{1}{24}\frac{(4\pi^2\alpha')^2}{32\pi^2}\Big[(R_T)_{\alpha\beta\gamma\delta}(R_T)^{\alpha\beta\gamma\delta} - 2(R_T)_{\alpha\beta}(R_T)^{\alpha\beta} - \\ - (R_N)_{\alpha\beta ij}(R_N)^{\alpha\beta ij} + 2\bar{R}_{ij}\bar{R}^{ij}\Big]\bigg) . \quad (1.2)$$

[1] We absorb $2\pi\alpha'$ into the U(1) gauge field strength.

[2] R-term does appear in the bosonic string D-brane action [15, 16].

Here $\alpha, \beta, \ldots = 0, 1, \ldots, p$ are the "parallel" and $i, j, \ldots = 1, \ldots, 9-p$ are the "transverse" indices and the tensors R_T and R_N are constructed from the world-volume and normal bundle connections and involve the second fundamental form (see [11, 13] for details). Note that (1.2) cannot be written just in terms of the curvature of the induced metric.

The analysis of [11, 13] excluded the standard R term but it did not address the possible presence of a Brans-Dicke type $Rf(X)$ terms, e.g., RX^2, where X stands for the "transverse" (X^i) scalar components and R is the curvature in the directions parallel to the brane.[3]

One may actually rule out the presence of similar terms with explicit X-dependence on quite general grounds. For example, given a supergravity soliton, its effective action may be derived in a static gauge. Since X^i is a goldstone boson reflecting the breaking of translational invariance, the action should depend on it only through ∂X.[4] Also, from the string-theory side, the U(1) gauge invariance combined with T-duality relation between $F_{\alpha\beta}$ and $\partial_\alpha X_i$ implies that the effective action reconstructed from string amplitudes should depend only on derivatives of the embedding coordinates. For example, we may start with the D9-brane action which does not contain transverse X-scalars; its action is $\int d^{10}\sigma\ e^{-\phi}[\sqrt{\det(G_{\alpha\beta}+F_{\alpha\beta})}+$ higher-derivative terms $+\ \alpha'^2 R^2$-terms $+\cdots]$. Assuming that the space-time metric is flat in some $9-p$ (toroidal) directions we may apply T-duality in these directions. That will effectively convert the corresponding components of A_α into X_i; we should then finish with a Dp-brane action in a curved metric in "parallel" directions, and there is no way it can contain $Rf(X)$ terms.

At this point one may wonder how the absence of the RX^2 term in D3-brane action is consistent with the AdS/CFT correspondence. One needs the standard conformal coupling term $\frac{1}{6}RX^2$ for the 6 scalars of the $N=4$ SYM theory to define the conformal stress tensor operator for the scalars, so one would expect this term to be present in a D3-brane probe action. This apparent puzzle was implicit already in [17]–[20].[5]

This puzzle was effectively resolved in [23]. As was explained there, starting with a *negative curvature Einstein space with a conformal boundary* with an arbitrary curved boundary metric $g_{\alpha\beta}$ (e.g., asymptotically AdS_{p+2} space), and considering a Dp-brane probe placed close and *parallel* to the boundary and described by the *standard* DBI action (1.1), one finds the effective conformal coupling term $\frac{p-2}{4(p-1)}R(g)r^2$ for the normal-direction scalar r ($r^2 = X_i X_i$ in the $AdS_5 \times S^5$ case). In more detail, expanding the

[3]One might wonder whether the previous analysis of curvature corrections was somehow incomplete. In [13] it was shown that the results of [11] had several ambiguities which had to be fixed before one could conclude that (1.2) is indeed the correct action to order α'^2. The case of the RX^2 term has a similar ambiguity as we shall discuss below.

[4]There is also a functional dependence of the background fields on $X = \bar{X} + \tilde{X}$, e.g., $G_{\mu\nu}(X) \simeq G_{\mu\nu}(\bar{X}) + \tilde{X}^i \partial_i G_{\mu\nu}(\bar{X}) + \ldots$, but it results only in terms with normal derivatives of the background fields which we are not interested in.

[5]One needs this curvature coupling term to argue [19] that the moduli space is lifted when SYM theory is defined on a sphere. This term is also crucial for conformally-invariant coupling of SYM theory to external conformal supergravity sources [20] needed to define the partition function form [21, 22] of the AdS/CFT correspondence.

metric $g_{\alpha\beta}$ in geodesic distance from the boundary[6] and plugging the expansion into the induced-metric determinant part of the DBI action for the brane, we get indeed the Rr^2 coupling between the boundary curvature and the normal scalar. It should be stressed that this curvature coupling term originates not from an additional contribution to the DBI action but rather from a specific choice of (the expansion of) the bulk metric, which is curved in both the boundary and the normal directions. The Weyl invariance of the normal scalar coupling to the curved boundary metric is again a consequence of the specific embedding of the brane in the AdS-type space with a conformal boundary.[7]

To complement the above (already convincing) arguments, we decided to explicitly verify the absence of the RX^2 term in the D-brane action by a first-principle — string S-matrix — computation. The previous discussions based on comparing the 3-point (one graviton h — two scalar X) amplitudes are not sufficient for this purpose. Here we are interested in the case when the gravitons are polarized along the D-brane world-volume directions.[8] In this case the RX^2 term gives vanishing contribution to hXX amplitude.[9]

The direct way to rule out the RX^2 term is then to compute the 2-graviton — 2-scalar ($hhXX$) 4-point superstring amplitude on the disc (with the Dp-brane boundary conditions) and to compare it with the field-theory amplitude predicted by the sum of the bulk Einstein action and the DBI action (1.1). This is what we are going to do below. We shall start with the superstring computation in sections 2 and 3 and then show in section 4 that the result is in complete agreement with the DBI action at the second-derivative order.

For completeness, let us briefly mention some other work on gravitational couplings on the brane. While there is no tree-level R-term on D-branes of type-II superstring theories, such term may be induced at 1-loop string level in the case of reduced amount of supersymmetry (see [24, 25]). In the case of D-branes in bosonic string (and non-BPS branes in superstring) in addition to tree-level R-term one expects to find $Rf(T)$ couplings for the open string tachyon T [15]. In the case of branes in AdS case (in the RS set-up [26]) one generically finds (by performing zero-mode analysis [27, 28] which should be related to the discussion in [23]) Brans-Dicke type terms on the 3-brane.

[6]It is crucial here that eq. (3.6) in [23] holds for the specific choices of the embedding and the metric. Then the radial coordinate r can be identified with the Riemann normal coordinate at the point we expand, and the expansion of the metric near the boundary will have the form $g_{\alpha\beta}(x,r) = g_{\alpha\beta}(x,0) + \frac{1}{3}r^2 R_{r\alpha r\beta} + \ldots$.

[7]In more general backgrounds or for different orientations of the brane one may end up with other effective couplings which may not be Weyl-invariant.

[8]In general, in the case of the "transversely" polarized gravitons, there is an ambiguity of adding terms like $(D_i\Omega^i_{\alpha\beta})g^{\alpha\beta}X^2$ and $(D^i D_\mu \xi^\nu_i)\Pi^\mu{}_\nu X^2$ where we have used the normal frame ξ^μ_i and the second fundamental form $\Omega^i_{\alpha\beta}$ ($\Pi^{\mu\nu} \equiv \partial_\alpha X^\mu \partial_\alpha X^\nu$). The bulk covariant derivatives are projected to the normal and tangent bundles using the projectors $\delta^{ij}\xi^\mu_i\xi^\nu_j$ and $\Pi^{\mu\nu}$ [11, 13]. Such terms with appropriate coefficients can cancel the contribution of RX^2 term to the hXX amplitude without any influence on the four-scalar amplitude.

[9]Notice also that the possible ambiguities mentioned in the previous footnote give individually vanishing contributions and do not lead to any contradiction.

2. Preliminaries

As explained above, we intend to compute the tree-level (disc) scattering amplitude involving two closed-string modes — gravitons polarized along the brane – and two open string modes – scalars X describing transverse fluctuations of the brane. This computation is aimed at determining whether a world-volume coupling RX^2 is present in the D-brane action. Notice that the contribution from such a term to the one graviton — two scalar amplitude is vanishing as a result of the lowest order equations of motion.

D-branes are non-perturbative string states [4]. This means that they are not part of the ordinary string spectrum and at weak coupling have infinite mass compared to string modes. In space-time D-branes are represented as static p-dimensional defects. As usual, that means that we must impose different (Neumann and Dirichlet) boundary conditions on the tangent and normal directions to the D-brane,

$$\begin{aligned} \partial_\perp X^\alpha \,|_{\partial\Sigma} &= 0 \\ X^i \,|_{\partial\Sigma} &= 0 \end{aligned} \tag{2.1}$$

The Greek indices ($\alpha = 0, 1 \ldots, p$), correspond to the coordinates parallel to the brane (we shall call them "world-volume" directions) and the Latin ones ($i = p+1, \ldots, 9$) — to the coordinates in the directions normal to the brane.

Before presenting details of our calculation, let us review the basic formalism of string vertex operators and their expectation values on the disc. We follow closely the review of [8] and references there, in particular [10, 7].

The string operators for an NS-NS massless closed string have the following general form

$$V(z,\bar z) = \epsilon_{\mu\nu} : V_s^\mu(z) :: V_s^\nu(\bar z) : \tag{2.2}$$

where $\mu = 0, 1, \ldots, 9$ and $s = 0, -1$ denotes the superghost charge or equivalently the picture in which the operator is defined. The total superghost charge on the disk is required to be $Q_{sg} = -2$ as a consequence of the super-diffeomorphism invariance. The holomorphic parts of the in the pictures 0 and 1 are given by:

$$\begin{aligned} V_{-1}^\mu(p,z) &= e^{-\phi(z)}\psi^\mu(z) e^{ip\cdot X(z)} \\ V_0^\mu(p,z) &= (\partial X^\mu(z) + ip\cdot\psi(z)\psi^\mu(z)) e^{ip\cdot X(z)} \end{aligned} \tag{2.3}$$

The Green's functions on the disc are found using the method of image charges on a two dimensional surface [31]. Each string vertex inserted at position z on the disc has an image at $1/\bar z$. Imposing Neumann or Dirichlet boundary conditions we find the correlators on the disc [29, 8, 30, 13]:

$$\begin{aligned} \langle \partial_z X^\mu(z) e^{i(Dp)\cdot X(\bar w)} \rangle &= \frac{i(Dp)^\mu \bar w}{1 - z\bar w} \\ \langle \partial_{\bar z} X^\mu(\bar z) e^{ip\cdot X(w)} \rangle &= \frac{i(Dp)^\mu w}{1 - \bar z w} \\ \langle \partial_z X^\mu(z) \partial_{\bar w} X^\nu(\bar w) \rangle &= \frac{\eta^{\mu\nu}}{(1 - z\bar w)^2} \end{aligned}$$

JHEP12(2002)001

$$
\begin{aligned}
\langle \psi^\mu(z)\psi^\nu(w)\rangle &= -\frac{\eta^{\mu\nu}}{z-w} \\
\langle \psi^\mu(z)\bar{\psi}^\nu(\bar{w})\rangle &= i\frac{D^{\mu\nu}}{1-z\bar{w}} \\
\langle c(z_1)c(z_2)c(z_3)\rangle &= C^{\text{ghost}}_{D_2}(z_1-z_2)(z_2-z_3)(z_1-z_3) \\
\langle c(z_1)c(z_2)\bar{c}(\bar{z}_3)\rangle &= C^{\text{ghost}}_{D_2}(z_1-z_2)(1-z_1\bar{z}_3)(1-z_2\bar{z}_3) \\
\langle e^{-\phi(z)}e^{-\bar{\phi}(\bar{w})}\rangle &= \frac{1}{1-z\bar{w}}
\end{aligned} \tag{2.4}
$$

where D^ν_μ reverses signs of the fields with Dirichlet conditions and is defined in [9] ($D^\lambda_\mu D_{\lambda\nu} = \eta_{\mu\nu}$). For the scattering of the "world-volume polarized" gravitons with the "transverse" scalars we have the following conditions on the momenta and polarizations due to lowest order equations of motion and momentum conservation:

$$
\begin{aligned}
k_1+k_2+p+q = 0\,, &\qquad \mathrm{Tr}(\epsilon_1) = \mathrm{Tr}(\epsilon_2) = 0 \\
\epsilon_1\cdot p = 0\,, &\qquad \epsilon_2\cdot q = 0
\end{aligned} \tag{2.5}
$$

$$
\zeta_n\cdot p = \zeta_n\cdot q = \zeta_n\cdot k_n = 0\,. \tag{2.6}
$$

Here $n = 1,2$ labels the two scalars with momenta k^μ_n and polarizations ζ^μ_n while graviton momenta are p and q and their traceless polarization tensors have non-zero components only along the world-volume directions of the brane. All momenta are assumed to have only "world-volume" components being non-vanishing.

3. String theory amplitude

The correlator we need to compute is:

$$
\begin{aligned}
A_{\text{string}} \propto \int_{|z|\le 1} d^2z \int_{|x|=1} dx\, &\langle c(z')\bar{c}(\bar{z}')V^{\mu\nu}_{(-1,-1)}(z',\bar{z}')V^{\rho\sigma}_{(0,0)}(z,\bar{z}) \times \\
&\times (c(x)-c(-x))V^i_{(0)}(x)V^j_{(0)}(-x)\rangle(\epsilon_1 D)_{\mu\nu}(\epsilon_2 D)_{\rho\sigma}\zeta^1_i\zeta^2_j
\end{aligned} \tag{3.1}
$$

where we set $z' = \bar{z}' = 0$ and we have chosen a gauge fixing of the position of the open string operators at opposite points on the circle $|z| = 1$. [10]

The correlators of the ghosts and superghosts are (using (2.4)):

$$
\begin{aligned}
\langle (c(x)-c(-x))c(0)\bar{c}(0)\rangle &= C^{\text{ghost}}_{D_2} 2x \\
\langle e^{-\phi(0)}e^{-\bar{\phi}(0)}\rangle &= 1
\end{aligned}
$$

Since the scalars have polarizations only in the transverse direction and all momenta are in world-volume directions the computation simplifies considerably. From $V^i_{(0)}V^j_{(0)}$ we need only to retain the following correlators:

$$
\langle \partial X^i \partial X^j\rangle = -\frac{\eta^{ij}}{(2x)^2}\,, \qquad \langle \psi^i\psi^j\rangle = -\frac{\eta^{ij}}{(2x)}\,, \tag{3.2}
$$

[10] The gauge fixing can be made more rigorous by changing the coordinates from x_1, x_2 for the scalars to $v_\pm = x_1 \pm x_2$ and then gauge fixing $v_+ = 0$.

JHEP12(2002)001

with all other correlators (coming from the cross product of world-sheet scalars and fermions in the $V^i_{(0)}, V^j_{(0)}$ vertices) giving vanishing contributions. It is easy to check that given the symmetries of the ϵ_1, ϵ_2 tensors the part in $V^{\mu\nu}_{(-1,-1)}(z', \bar z')\, V^{\rho\sigma}_{(0,0)}(z, \bar z)$ multiplying $\langle \partial X^i \partial X^j\rangle$ in (3.2) is vanishing. It remains then to compute the following correlator

$$\begin{aligned}&\Big\langle k_1\cdot\psi(x) k_2\cdot\psi(x)\Big[\psi^\alpha(0)\bar\psi^\beta(0)\Big(\partial X^\gamma\bar\partial X^\delta + i\partial X^\gamma q\cdot\bar\psi\bar\psi^\delta + i\bar\partial X^\delta q\cdot\psi\psi^\gamma - q\cdot\psi\psi^\gamma q\cdot\bar\psi\bar\psi^\delta\Big)\Big]\times\\ &\quad\times\Big[e^{2ik_1X}e^{2ik_2X}e^{ipX}e^{ip\bar X}e^{iqX}e^{iq\bar X}\Big]\Big\rangle \end{aligned}\tag{3.3}$$

which should be multiplied with $\langle\psi^i\psi^j\rangle$ in (3.2) and $(\epsilon_1 D)_{\mu\nu} \to (\epsilon_1)_{\alpha\beta}$, $(\epsilon_2 D)_{\rho\sigma} \to (\epsilon_2)_{\gamma\delta}$.

After some tedious computations and using the symmetries of the polarization tensors and the symmetry under interchange of the two scalars we arrive at the following result for the integrand in (3.1) (we ignore overall numerical coefficient):

$$\begin{aligned}\mathcal{A}_{\rm string} \sim\ & i|1-\bar x z|^{4q\cdot k1}|1-x\bar z|^{4q\cdot k2}|z|^{2p\cdot q}2^{4k_1\cdot k_2}x^{-1}\eta^{ij}\times\\ &\times\bigg(k_1^\alpha k_2^\beta\Big[k_1^\gamma k_1^\delta I_1 + k_1^\gamma k_2^\delta(2I_2-1)\Big] - k_1^\alpha k_1^\gamma\left(k_2 q\eta^{\beta\delta} - k_2^\delta q^\beta\right)(2I_2-1) -\\ &\quad - \frac{k_1^\alpha k_2^\gamma}{2}\left(k_2 q\eta^{\beta\delta} - k_2^\delta q^\beta\right) I_3 - \frac{k_1^\gamma k_2^\alpha}{2}\left(k_1 q\eta^{\beta\delta} - k_1^\delta q^\beta\right) I_1 +\\ &\quad + \frac{1}{2}\left(k_1 q\eta^{\alpha\gamma} - k_1^\gamma q^\alpha\right)\left(k_2 q\eta^{\beta\delta} - k_2^\delta q^\beta\right) I_2\bigg) + (1\leftrightarrow 2)\end{aligned}\tag{3.4}$$

where $(1 \leftrightarrow 2)$ stands for symmetrization under interchange of the two scalars and the two graviton polarizations and momenta and

$$I_1 = \frac{|1+\bar x z|^2}{|1-\bar x z|^2|z|^2} \qquad I_2 = \frac{\left(1-|z|^2\right)^2}{|1+\bar x z|^2|1-\bar x z|^2|z|^2} \qquad I_3 = \frac{|1-\bar x z|^2}{|1+\bar x z|^2|z|^2}\,. \tag{3.5}$$

Next, we should perform the triple integration over the world-sheet coordinates x and $z, \bar z$. This may seem a difficult task a priori but there is a change of coordinates that allows one to decouple the x and $z, \bar z$ integrals.[11] The x-integration is trivial — we are left with the simple contour integral $\oint dx/x = 2\pi i$. The remaining z-integral should be possible to express in terms of the hypergeometric functions.[12]

Since we are interested only in determining the RX^2 contribution, we do not actually need to compute the whole integral: it is sufficient to extract the terms with only two powers of the momenta. Given that all the terms in (3.4) have four powers of momenta we should just extract the residues of the poles on the disc $|z| \le 1$. The integrand has poles at $z = 0, 1, -1$.

To extract the poles, it is useful first to conformally map the disc to the left half complex plane using:

$$v = \frac{z-i}{z+i}\,, \tag{3.6}$$

[11] If we set $z = |z|e^{i\theta}$ and $x = e^{i\varphi}$ then a shift of the angle $\theta \to \theta - \varphi$ amounts to the change of coordinates $z \to z' = z\bar x$ and then the integrals over z' and x separate.

[12] This should be possible following the discussion of similar integrals in [34, 35]. For string amplitudes on the sphere one can go to the conformally equivalent complex plane and factorize the double integral into two contour ones. However, the presence of the boundary (real line boundary of the upper half of the complex plane) is a complication, and we have not been able to compute the integral explicitly.

JHEP12(2002)001

so that the poles at $z = 0, 1, -1$ are mapped to $v = -1, i, -i$. The measure d^2z transforms to $d^2v\ |v-1|^4$. We need to keep in mind now that since we are integrating over the left half plane the contribution of the poles $\pm i$ should be taken with factor $1/2$ since they are at the boundary of the integration region. Then we express each of the integrands in (3.5) in terms of v and then expand them in some small region around each pole. The residue is extracted using the standard rule (see, e.g., [32], eq (4.7)):

$$\int dx\ x^{-1+\alpha' k_i k_j} \to -\frac{1}{\alpha' k_i k_j}\,. \tag{3.7}$$

For example, let us consider I_1 in (3.5). After the conformal transformation to the left half plane

$$I_1 = \frac{|1+z|^2}{|1-z|^2|z|^2} \quad \to \quad \frac{|1-v|^2|v-i|^2}{|1+v|^2|v+i|^2} \tag{3.8}$$

it has poles at $v = -i, -1$. Expanding around a given pole both I_1 and the terms in the first line of (3.4) we get:

$$\frac{1}{2}\int dx\, x \frac{1}{|-i-1|^4}\frac{4}{x^2}x^{4qk_1} \to \frac{1}{8qk_1}\,, \qquad \int dx\, x\frac{1}{|-1-1|^4}\frac{4}{x^2}x^{2pq} \to \frac{1}{8pq}\,, \tag{3.9}$$

where we accounted for the relative factor $1/2$ between poles at the boundary and in the interior and also included the d^2v contribution around the pole as a measure for a small disc of radius x in polar coordinates.

Following the same procedure for all terms in the amplitude and symmetrizing with respect to the two scalars we get:

$$\begin{aligned} A_{\text{string}} \sim (k_1\epsilon_1k_2)&\left[-(k_1\epsilon_2k_1)\frac{qk_2}{4(qk_1)(pq)}-(k_2\epsilon_2k_2)\frac{qk_1}{4(qk_2)(pq)}+(k_1\epsilon_2k_2)\frac{1}{2pq}\right]-\\ &-\frac{1}{4pq}\left[(k_1\epsilon_1\epsilon_2k_1)(k_2q)+(k_2\epsilon_1\epsilon_2k_2)(k_1q)-(k_1\epsilon_1q)(k_1\epsilon_2k_2)-(k_2\epsilon_1q)(k_1\epsilon_2k_2)\right]+\\ &+\frac{k_1q}{4(k_2q)(pq)}\left[(k_1\epsilon_1\epsilon_2k_2)(k_2q)-(k_1\epsilon_1q)(k_2\epsilon_2k_2)\right]+\\ &+\frac{k_2q}{4(k_1q)(pq)}\left[(k_2\epsilon_1\epsilon_2k_1)(k_1q)-(k_2\epsilon_1q)(k_1\epsilon_2k_1)\right]+\\ &+\frac{1}{4pq}\Big[\mathrm{Tr}(\epsilon_1\epsilon_2)(k_1q)(k_2q)-(q\epsilon_1\epsilon_2k_2)(k_1q)-\\ &\qquad -(q\epsilon_1\epsilon_2k_1)(k_2q)+(q\epsilon_1q)(k_1\epsilon_2k_2)\Big]+(1\leftrightarrow 2) \end{aligned} \tag{3.10}$$

where $(1 \leftrightarrow 2)$ stands for the remaining symmetrization in graviton polarization and momenta. The apparent "double-pole" momentum factors ($\frac{1}{(qk_1)(pq)}$, etc.) here can be eliminated by using momentum conservation.

As a check, one may demonstrate that the result is gauge invariant under $(\epsilon_i)_{\mu\nu} \to (\xi_i)_\mu p_\nu + (\xi_i)_\nu p_\mu$ (the first and fourth lines of (3.10) each are separately gauge invariant, while the second and third lines combine into a gauge-invariant expression).

JHEP12(2002)001

Using conservation of momentum to eliminate some q-momenta in terms of k_1 and k_2, and symmetrizing in the graviton polarizations, we arrive at a much simpler expression:

$$
\begin{aligned}
A_{\text{string}} \sim 2(\zeta_1\zeta_2)\Bigg[&(k_1\epsilon_1k_2)(k_1\epsilon_2k_2)\frac{1}{2pq}+(k_1\epsilon_1\epsilon_2k_2)\frac{k_1q}{2(pq)}+(k_2\epsilon_1\epsilon_2k_1)\frac{k_2q}{2(pq)}+\\
&+(k_1\epsilon_1k_1)(k_2\epsilon_2k_2)\frac{k_1q}{4(k_2q)(pq)}+(k_2\epsilon_1k_2)(k_1\epsilon_2k_1)\frac{k_2q}{4(k_1q)(pq)}+\\
&+\mathrm{Tr}(\epsilon_1\epsilon_2)\frac{(k_2q)(k_1q)}{4(pq)}\Bigg]
\end{aligned}
\tag{3.11}
$$

4. D-brane field theory amplitude

Let us now compare the string-theory result with field theory amplitude that follows from graviton–scalar interaction terms in D-brane action.

The full field-theory action contains the standard bulk supergravity action (written in the Einstein frame)

$$
S_{10} = -\frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 + \cdots\right], \tag{4.1}
$$

and the D-brane action. The low energy dynamics of a Dp-brane in a curved space is encoded in the DBI action. The D-brane action contains the world-volume massless scalar fields $X^\mu(\sigma)$ (the embedding coordinates of the p-brane in the ambient space-time) and the U(1) gauge fields $A_\alpha(\sigma)$ coupled to the bulk supergravity fields (σ^α are the world-volume coordinates). In addition, there are supersymmetric partners of these fields but they will be irrelevant for our present discussion. We shall also ignore the WZ type term describing well-known coupling of D-brane to the R-R potentials. In terms of the Einstein-frame bulk metric $g_{\mu\nu}$ the DBI action may be written as

$$
S_{\text{DBI}} = -T_p\int d^{p+1}\sigma e^{\frac{p-3}{4}\phi}\sqrt{-\det\left[\tilde{g}_{\alpha\beta}(X) + e^{-\phi/2}\tilde{B}_{\alpha\beta}(X) + e^{-\phi/2}F_{\alpha\beta}\right]}, \tag{4.2}
$$

where $\tilde{g}_{\alpha\beta} = g_{\mu\nu}(X)\partial_\alpha X^\mu\partial_\beta X^\nu$, $\tilde{B}_{\alpha\beta} = B_{\mu\nu}(X)\partial_\alpha X^\mu\partial_\beta X^\nu$ are the pull-backs on the brane of the corresponding bulk tensors.

Our aim is to compute the 2-graviton (h)– 2-scalar (X) tree amplitude and to compare it with string-theory result, to see if we need to introduce some additional second-derivative interactions to the DBI action like RX^2.

The field-theory amplitude will contain contact $hhXX$ contribution from the DBI action, a scalar exchange between the two hXX vertices, and also a graviton exchange between the closed string vertex hhh present in the bulk supergravity action and the hXX vertex present in the DBI action (see figure1). Since there is no bulk coupling for the dilaton of the form ϕhh there will be no dilaton exchange diagram, i.e. the dilaton (and $B_{\mu\nu}$) contributions may be ignored in the present case. The scalar–gauge field couplings are also irrelevant, i.e. we may set $A_\alpha = 0$.

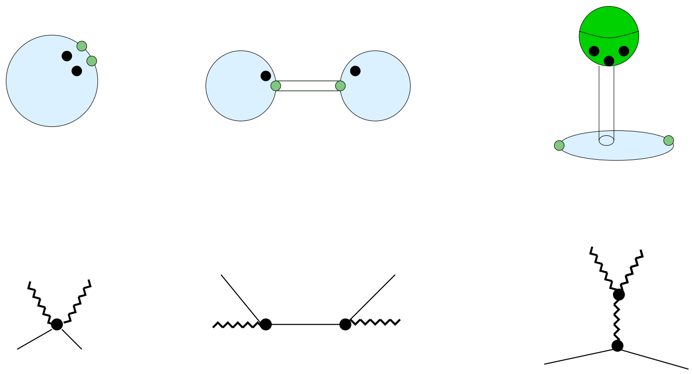

Figure 1: This figure shows three types of contributions to the scattering amplitude, both in string-theory (upper raw) and field-theory (lower raw) representation. In string diagrams darker dots stand for the graviton vertices and lighter dots for the scalar vertices; the double line is an open or closed string propagator. In field-theory diagrams the wiggly lines are graviton and straight lines are scalar propagators. The first contribution is a contact term (coming from the region when all 4 points on the disc are close to each other). The second one is the s-channel exchange contribution originating from the factorization of the disc into two discs connected by a scalar propagator (corresponding to the region where points come close pairwise). The third diagram corresponds to the region where the points of graviton insertions are close to each other so that the amplitude factorizes to a sphere and a disc connected by a graviton propagator.

Expanding the induced metric in (4.2) near the flat space and using the static gauge $X^\alpha = \sigma^\alpha$ we get ($\alpha = 0, \ldots, p;\ i = 1, \ldots, 9-p$)

$$\tilde{g}_{\alpha\beta} = g_{\alpha\beta}(\sigma, X) + 2g_{i(\alpha}(\sigma, X)\partial_{\beta)}X^i + g_{ij}(\sigma, X)\partial_\alpha X^i \partial_\beta X^j \tag{4.3}$$

$$g_{\alpha\beta} = \eta_{\alpha\beta} + 2\kappa h_{\alpha\beta}(\sigma, X)\,, \qquad g_{ij} = \delta_{ij} + 2\kappa h_{ij}(\sigma, X)\,, \qquad g_{\alpha i} = 2\kappa h_{\alpha i}(\sigma, X)\,. \tag{4.4}$$

In what follows we shall assume that the graviton is polarized parallel to the brane, i.e. $h_{ij} = 0 = h_{\alpha i}$.

To expand the square root of the determinant in (4.2) we use formula

$$\begin{aligned}\sqrt{\det(\delta^\alpha_\beta + M^\alpha_\beta)} &= 1 + \frac{1}{2}M^\alpha_\alpha - \frac{1}{4}M^\alpha_\beta M^\beta_\alpha + \frac{1}{8}\left(M^\alpha_\alpha\right)^2 + \\ &\quad + \frac{1}{6}M^\alpha_\beta M^\beta_\gamma M^\gamma_\alpha - \frac{1}{8}M^\alpha_\beta M^\beta_\alpha M^\gamma_\gamma + \frac{1}{48}\left(M^\alpha_\alpha\right)^3 \ldots\end{aligned} \tag{4.5}$$

Using Fourier representation for the fluctuations $h_{\alpha\beta}$ and X^i and expanding (4.2) we get the following scalar propagator and hXX and $hhXX$ vertices (multiplying them by i)

$$P_i^j = -\frac{i\delta_i^j}{k^2} \tag{4.6}$$

$$V^h_{XX} = -2i\kappa T_p(\zeta_1\zeta_2)(k_1\epsilon k_2) \tag{4.7}$$

$$V^{hh}_{XX} = 4i\kappa^2 T_p(\zeta_1\zeta_2)\left[(k_1\epsilon_1\epsilon_2 k_2) + (k_2\epsilon_1\epsilon_2 k_1) - \frac{1}{2}(k_1k_2)\mathrm{Tr}(\epsilon_1\epsilon_2)\right]. \tag{4.8}$$

JHEP12(2002)001

To compute the graviton exchange contribution we will need also the hXX vertex with off-shell graviton (which we shall denote by $H_{\mu\nu}$, with polarization tensor $E_{\mu\nu}$)

$$V_{XX}^{H} = -2i\kappa T_p\left[(\zeta_1\zeta_2)(k_1 E k_2)(\zeta_1\zeta_2) - \frac{1}{2}(\zeta_1\zeta_2)(k_1 k_2)E^{\mu}_{\mu} - (k_1 k_2)(\zeta_1 E\zeta_2)\right]. \tag{4.9}$$

The graviton propagator corresponding to the bulk Einstein action is

$$(P_H)_{\mu\nu,\lambda\rho} = -\frac{i}{2p^2}\left(\eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda} - \frac{1}{4}\eta_{\mu\nu}\eta_{\lambda\rho}\right), \tag{4.10}$$

while the vertex for the two on-shell gravitons and one off-shell one is [33]:

$$\begin{aligned} V(H,h_1,h_2) &= -4\kappa\left(H_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}H^{\lambda}_{\lambda}\right)\left[h_{\rho\sigma}R^{(1)}_{\rho\mu\sigma\nu} - \frac{1}{4}h_{\rho\sigma,\mu}h_{\rho\sigma,\nu} + \right.\\ &\qquad\qquad \left. + \frac{1}{2}h_{\rho\mu,\sigma}(h_{\sigma\nu,\rho} - h_{\rho\nu,\sigma})\right],\\ R^{(1)}_{\rho\mu\sigma\nu} &= \frac{1}{2}\left(h_{\nu\rho,\mu\sigma} + h_{\mu\sigma,\nu\rho} - h_{\mu\nu,\sigma\rho} - h_{\rho\sigma,\nu\mu}\right). \end{aligned} \tag{4.11}$$

The relevant field-theory amplitude contributions are shown in figure 1. The contact contribution is given by the V_{XX}^{hh} term in (4.8):

$$(A_c)_{XX}^{hh} = i\kappa^2 T_p(\zeta_1\zeta_2)\left[4(k_1\epsilon_1\epsilon_2 k_2) + 4(k_2\epsilon_1\epsilon_2 k_1) - 2(k_1 k_2)\mathrm{Tr}(\epsilon_1\epsilon_2)\right]. \tag{4.12}$$

The contribution from the exchange of a scalar field (s-channel) is:

$$(A_s)_{XX}^{hh} = (V^i)_{XX}^{h}P_i^j(V_j)_{XX}^{h} = 2i\kappa^2 T_p\left[(k_1\epsilon_1 k_1)(k_2\epsilon_2 k_2)\frac{1}{k_2 q} + (k_1\epsilon_2 k_1)(k_2\epsilon_1 k_2)\frac{1}{k_1 q}\right], \tag{4.13}$$

where we have symmetrized in both scalars and gravitons. Finally, it is long but straightforward to compute the contribution from the exchange of a graviton (t-channel) using (4.9), (4.10) and (4.11):[13]

$$\begin{aligned} (A_t)_{XX}^{hh} &= (V^{\mu\nu})_{XX}^{H}(P_H)_{\mu\nu,\lambda\rho}V^{\lambda\rho}(H,h_1,h_2)\\ &= i\kappa^2 T_p\Big[-2(k_1\epsilon_1\epsilon_2 k_2)(\frac{k_1 q}{pq}+1) - 2(k_2\epsilon_1\epsilon_2 k_1)(\frac{k_2 q}{pq}+1) - \mathrm{Tr}(\epsilon_1\epsilon_2)\frac{(k_1 q)(k_2 q)}{pq} -\\ &\qquad - (k_1\epsilon_1 k_1)(k_2\epsilon_2 k_2)\frac{k_1 q}{(k_2 q)(pq)} - (k_1\epsilon_2 k_1)(k_2\epsilon_1 k_2)\frac{k_2 q}{(k_1 q)(pq)} -\\ &\qquad - \frac{1}{pq}(k_1\epsilon_2 k_2)\left((k_1\epsilon_1 k_1) + 2(k_1\epsilon_1 k_2) + (k_2\epsilon_1 k_2)\right) +\\ &\qquad + \frac{1}{pq}(k_1\epsilon_1 k_2)((k_2\epsilon_2 k_2) + (k_1\epsilon_2 k_1)) + (\epsilon_1 \leftrightarrow \epsilon_2)\Big]. \end{aligned} \tag{4.14}$$

Combining (4.12), (4.13) and (4.14) and using the symmetry under $\epsilon_1 \leftrightarrow \epsilon_2$ in $(A_t)_{XX}^{hh}$ to simplify some terms, it is possible to show that the field theory amplitude reproduces the string-theory amplitude (3.11). This rules out the presence of an extra RX^2 term in the DBI action.

[13] It turns out that the last term in V_{XX}^{H} does not actually contribute due to the structure of the three-graviton vertex.

JHEP12(2002)001

The only potential caveat could be the following. Since we have not been careful to include the normalization factors in the string amplitude, the agreement is only up to the overall coefficient. One could imagine that RX^2 term could produce a contribution which is also proportional to the full string amplitude (3.11). However, this possibility is excluded as RX^2 cannot give an s-channel contribution present in the string amplitude.[14] One may also try to add some other terms which could account for an additional s-channel contribution; the only candidate with the right number of momenta is $X_i \Omega^{i\alpha}_{\alpha}$, where $\Omega^{i}_{\alpha\beta}$ is the second fundamental form [11, 13]. Such term, however, is proportional to the lowest-order equations of motion and therefore can be removed by a field redefinition.

We conclude that the standard DBI action (4.2) is in full agreement with the string S-matrix at the second derivative order.

Acknowledgments

We would like to thank S. Kachru for raising the question about possible presence of RX^2 term in the D-brane action which motivated the present work. We are also grateful to I. Klebanov for extensive discussions and collaboration at an initial stage of this project. We also thank P.Vanhove for a useful advice. A.A.T. acknowledges the hospitality of the Aspen Center for Physics while this work was in progress. The work of A.A.T. was supported in part by the grants DOE DE-FG02-91ER40690, PPARC SPG 00613, INTAS 99-1590, and by the Royal Society Wolfson Research Merit Award. The work of A.F. was supported in part by the European Commission under contracts HRPN-CT-2000-00131 and HRPN-CT-2000-00148.

References

[1] E.S. Fradkin and A.A. Tseytlin, *Nonlinear electrodynamics from quantized strings*, *Phys. Lett.* **B 163** (1985) 123.

[2] A. Abouelsaood, C.G. Callan, C.R. Nappi and S.A. Yost, *Open strings in background gauge fields*, *Nucl. Phys.* **B 280** (1987) 599.

[3] J. Dai, R.G. Leigh and J. Polchinski, *New connections between string theories*, *Mod. Phys. Lett.* **A 4** (1989) 2073.

[4] J. Polchinski, *Dirichlet-branes and Ramond-Ramond charges*, *Phys. Rev. Lett.* **75** (1995) 4724 [hep-th/9510017].

[5] R.G. Leigh, *Dirac- Born-Infeld action from dirichlet sigma model*, *Mod. Phys. Lett.* **A 4** (1989) 2767.

[6] O.D. Andreev and A.A. Tseytlin, *Partition function representation for the open superstring effective action: cancellation of mobius infinities and derivative corrections to Born-Infeld lagrangian*, *Nucl. Phys.* **B 311** (1988) 205.

[7] S.S. Gubser, A. Hashimoto, I.R. Klebanov and J.M. Maldacena, *Gravitational lensing by p-branes*, *Nucl. Phys.* **B 472** (1996) 231 [hep-th/9601057].

[14] It may still contribute to the t-channel since it modifies the V^{H}_{XX} vertex.

[8] A. Hashimoto and I.R. Klebanov, *Scattering of strings from D-branes*, *Nucl. Phys.* **55B** *(Proc. Suppl.)* (1997) 118 [hep-th/9611214].

[9] M.R. Garousi and R.C. Myers, *Superstring scattering from D-branes*, *Nucl. Phys.* **B 475** (1996) 193 [hep-th/9603194].

[10] M.R. Garousi and R.C. Myers, *World-volume interactions on D-branes*, *Nucl. Phys.* **B 542** (1999) 73 [hep-th/9809100].

[11] C.P. Bachas, P. Bain and M.B. Green, *Curvature terms in D-brane actions and their M-theory origin*, *J. High Energy Phys.* **05** (1999) 011 [hep-th/9903210].

[12] N. Wyllard, *Derivative corrections to D-brane actions with constant background fields*, *Nucl. Phys.* **B 598** (2001) 247 [hep-th/0008125]; *Derivative corrections to the D-brane Born-Infeld action: non-geodesic embeddings and the seiberg-Witten map*, *J. High Energy Phys.* **08** (2001) 027 [hep-th/0107185].

[13] A. Fotopoulos, *On α'^2 corrections to the D-brane action for non- geodesic world-volume embeddings*, *J. High Energy Phys.* **09** (2001) 005 [hep-th/0104146].

[14] A.A. Tseytlin, *Heterotic - type-I superstring duality and low-energy effective actions*, *Nucl. Phys.* **B 467** (1996) 383 [hep-th/9512081].

[15] S. Corley, D.A. Lowe and S. Ramgoolam, *Einstein-hilbert action on the brane for the bulk graviton*, *J. High Energy Phys.* **07** (2001) 030 [hep-th/0106067].

[16] F. Ardalan, H. Arfaei, M.R. Garousi and A. Ghodsi, *Gravity on noncommutative D-branes*, hep-th/0204117.

[17] S.S. Gubser, I.R. Klebanov and A.A. Tseytlin, *String theory and classical absorption by three-branes*, *Nucl. Phys.* **B 499** (1997) 217 [hep-th/9703040].

[18] S.S. Gubser and I.R. Klebanov, *Absorption by branes and schwinger terms in the world volume theory*, *Phys. Lett.* **B 413** (1997) 41 [hep-th/9708005].

[19] E. Witten, *Anti-de Sitter space, thermal phase transition and confinement in gauge theories*, *Adv. Theor. Math. Phys.* **2** (1998) 505 [hep-th/9803131].

[20] H. Liu and A.A. Tseytlin, *$D = 4$ super Yang-Mills, $D = 5$ gauged supergravity and $D = 4$ conformal supergravity*, *Nucl. Phys.* **B 533** (1998) 88 [hep-th/9804083].

[21] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109].

[22] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[23] N. Seiberg and E. Witten, *The $D1/D5$ system and singular CFT*, *J. High Energy Phys.* **04** (1999) 017 [hep-th/9903224].

[24] E. Kiritsis, N. Tetradis and T.N. Tomaras, *Thick branes and 4d gravity*, *J. High Energy Phys.* **08** (2001) 012 [hep-th/0106050]; *Induced gravity on RS branes*, *J. High Energy Phys.* **0203** (2002) 019 [hep-th/0202037].

[25] I. Antoniadis, R. Minasian and P. Vanhove, *Non-compact Calabi-Yau manifolds and localized gravity*, hep-th/0209030.

[26] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221]; *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[27] J. Garriga and T. Tanaka, *Gravity in the brane-world*, *Phys. Rev. Lett.* **84** (2000) 2778 [hep-th/9911055].

[28] T. Chiba, *Scalar-tensor gravity in two 3-brane system*, *Phys. Rev.* **D 62** (2000) 021502 [gr-qc/0001029].

[29] J. Polchinski, *String theory.* vol. **1**: *An introduction to the bosonic string*, Cambridge Univ. Press, .1998.

[30] I.R. Klebanov and L. Thorlacius, *The size of p-branes*, *Phys. Lett.* **B 371** (1996) 51 [hep-th/9510200].

[31] C.P. Burgess and T.R. Morris, *Open superstrings a la polyakov*, *Nucl. Phys.* **B 291** (1987) 285.

[32] Z. Bern, *String based perturbative methods for gauge theories*, hep-ph/9304249.

[33] M.T. Grisaru, P. van Nieuwenhuizen and C.C. Wu, *Gravitational born amplitudes and kinematical constraints*, *Phys. Rev.* **D 12** (1975) 397.

[34] H. Kawai, D.C. Lewellen and S.H.H. Tye, *A relation between tree amplitudes of closed and open strings*, *Nucl. Phys.* **B 269** (1986) 1.

[35] V. Dotsenko, *Serie de Cours sur la Theorie Conforme*, LPTHE